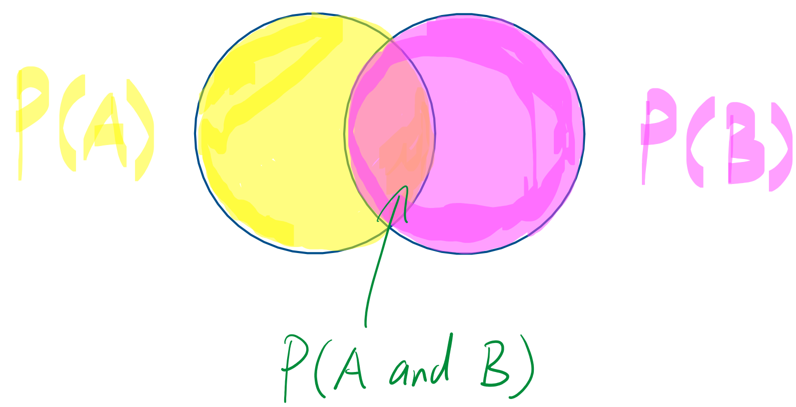
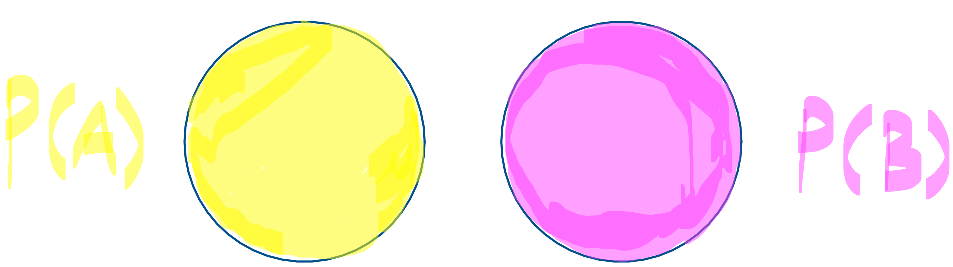
X. Addition $\cup \leftarrow$ or, it includes 'AND'

The addition rule in probability can be express as P(A or B). It is the probability that event A occur or event B occur, as a single outcome of a procedure. It is an event that compose of 2 or more events.



$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

If event A and event B can not occur at the same time, then it is called mutually exclusive or disjoint.



$P(A \text{ and } B) = 0$

Contingency Table

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Total	9	13	3	25

The **intersection** of **Varsity Team row AND the Guards column contains a** 3

There are 3 players who are both a **Varsity player AND a Guard**

The **intersection** of the **Varsity Team row AND the Forward column contains a** 5

There are 5 players who are both a **Varsity player AND a Forward**

The **intersection** of the **Varsity Team row AND the Centers column contains a** 2

There are 2 players who are both a **Varsity player AND a Center**

The **intersection** of the **Jr. Varsity Team row AND the Guards column contains a** 6

There are 6 players who are both a **Jr. Varsity player AND a Guard**

The **intersection** of the **Jr. Varsity Team row AND the Forward column contains a** 8

There are 8 players who are both a **Jr. Varsity player AND a Forward**

The **intersection** of the **Jr. Varsity Team row AND the Centers column contains a** 1

There is 1 player who is both a **Jr. Varsity player AND a Center**

eg

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Total	9	13	3	25

The entire row of Jr. Varsity and the entire column of Forward are blue so the answer is

Jr. Varsity Team OR Forward but you could also use **Forward OR Jr. Varsity Team**

Forwards: $5 + 8 = 13$ ✓

Jr. Varsity Team: $6 + 8 + 1 = 15$ ✓

eg

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Total	9	13	3	25

The entire row of Varsity and the entire column of Center are blue so the answer is

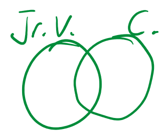
Varsity Team OR Center you could also use **Center OR Varsity Team**

2 has count twice

eg

P(Jr. Varsity **OR** Center)

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Total	9	13	3	25



P(Jr. Varsity or Center) = P(Jr. Varsity) + P(Center) − P(Jr. Varsity and Center)

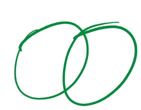
$= \frac{15}{25} + \frac{3}{25} - \frac{1}{25}$

$= \boxed{\frac{17}{25}}$

eg

P(Forward **OR** Center)

	Guards	Forwards	Centers	Total
Varsity Team	3	5	2	10
Jr. Varsity Team	6	8	1	15
Total	9	13	3	25

 now 

P(Forward or Center) = P(Forward) + P(Center) − P(Forward and Center)

= 13 + 3 0 ← nobody

$$= \frac{13}{25} + \frac{3}{25} - \frac{0}{25} \leftarrow \text{nobody}$$

$$= \boxed{\frac{16}{25}}$$

Eg. Find probability that a randomly selected person is a math major OR a female?

	Math	English	Total
Female	10	20	30
Male	4	67	71
Total	14	87	101

Sol:

$$P(\text{math or female}) = P(\text{math}) + P(\text{female}) - P(\text{math and female})$$

$$= \frac{14}{101} + \frac{30}{101} - \frac{10}{101}$$

$$= \boxed{\frac{34}{101}}$$

Eg. Given the fast food serving information below:

	McDonald's	Burger King	Wendy's	Taco Bell	
Orders accurate	329	264	249	145	987
Orders not accurate	33	54	31	13	131
	362	318	280	158	1118

a. If one order is selected, find the probability of getting an order that is not accurate.
b. If one order is selected, find the probability of getting an order that is from Wendy's or not accurate?

S: Missing totals ...

a. $P(\text{not accurate}) = \frac{131}{1118} \approx \boxed{0.12}$

b. $P(\text{Wendy's or not accurate}) = P(\text{Wendy's}) + P(\text{not accurate}) - P(\text{Wendy's and not accurate})$

and not accurate)

$$= \frac{280}{1118} + \frac{131}{1118} - \frac{31}{1118}$$

$$= \frac{380}{1118}$$

$$\approx \boxed{0.34}$$

eg

The table below describes the smoking habits of a group of asthma sufferers. If one of the 1156 people is randomly selected, find the probability that the person is a man or a heavy smoker. Round to three decimal places as needed.

	Nonsmoker	Occasional Smoker	Regular Smoker	Heavy Smoker	Total
Men	431	50	71	49	601
Women	382	48	86	39	555
Total	813	98	157	88	1156

S:

$$P(\text{men or heavy smoker}) = P(\text{men}) + P(\text{heavy smoker}) - P(\text{men and heavy smoker})$$

$$= \frac{601}{1156} + \frac{88}{1156} - \frac{49}{1156}$$

$$\approx \boxed{0.554}$$

XI. Conditional Probability ← keyword: given ← A.I.

A conditional probability is when the probability of event A occurs, given that event B has happened. It denotes $P(A \mid B)$.

$$\boxed{P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}} \leftarrow \frac{\text{"and"}}{\text{RHS}}$$

$$\boxed{P(A|B) = \frac{P(A \text{ and } B)}{P(B)}} \leftarrow \frac{\text{"and"}}{\text{RHS}}$$

Note: It means what is the prob of A, if B has already happened?

eg.

Find the following using the table:

a. If 1 of the 555 test subjects is randomly selected, find the probability that the subject had a positive test result, given that the subject actually uses drugs. That is, find *P*(positive test result | subject uses drugs).

	Positive Test Result (Test shows drug use.)	**Negative Test Result** (Test shows no drug use.)	
Subject Uses Drugs	45 (True Positive)	5 (False Negative)	50
Subject Does Not Use Drugs	25 (False Positive)	480 (True Negative)	505
	70	485	555 ✓

S:

$P(\text{positive test result} \mid \text{subject uses drugs})$

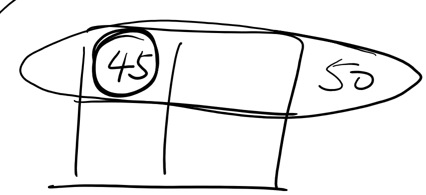
$$= \frac{P(\text{positive test result} \mid \text{subject uses drugs})}{P(\text{subject uses drugs})} \swarrow \frac{\text{"and"}}{\text{RHS}}$$

$$= \frac{\frac{45}{555}}{\frac{50}{555}} \rightarrow \frac{45}{\cancel{555}} \cdot \frac{\cancel{555}}{50}$$

$$= \frac{45}{50}$$

$$= \boxed{0.9}$$

Short-cut =



$$\frac{45}{50}$$

$$= \boxed{0.9}$$

eg

Find the following using the table:

b. If 1 of the 555 test subjects is randomly selected, find the probability that the subject actually uses drugs, given that he or she had a positive test result. That is, find *P*(subject uses drugs | positive test result).

	Positive Test Result (Test shows drug use.)	**Negative Test Result** (Test shows no drug use.)
Subject Uses Drugs	45 (True Positive)	5 (False Negative)
Subject Does Not Use Drugs	25 (False Positive)	480 (True Negative)

S: $P(\text{subject uses drugs} \mid \text{positive test result})$

$$= \frac{P(\text{subject uses drugs and positive test result})}{P(\text{positive test result})}$$

$$= \frac{45}{70}$$

$$\approx \boxed{0.64}$$

45	
25	
70	

Eg. The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

a) Has a speeding ticket given they have a red car

b) Has a red car given they have a speeding ticket

	Speeding ticket	No speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

S: a) $P(\text{speeding ticket} \mid \text{red car})$

$$= \frac{P(\text{speeding ticket and red car})}{P(\text{red car})}$$

$$= \frac{15}{150}$$

$$= \boxed{0.1}$$

b) $P(\text{red car} \mid \text{speeding ticket})$

$$= \frac{P(\text{red car and speeding ticket})}{P(\text{speeding ticket})}$$

$$= \frac{15}{60}$$

$$= \boxed{0.25}$$

Eg. A home pregnancy test was given to women, then pregnancy was verified through blood tests. The following table shows the home pregnancy test results. Find

a) P(not pregnant | positive test result)

b) P(positive test result | not pregnant)

	Positive test	Negative test	Total
Pregnant	70	4	74
Not Pregnant	5	14	19
Total	75	18	93

a) $= \dfrac{P(\text{not pregnant and positive test result})}{P(\text{positive test result})}$

$= \dfrac{5}{75}$

$\approx \boxed{0.067}$

b) $= \dfrac{P(\text{positive test result and not pregnant})}{P(\text{not pregnant})}$

$= \dfrac{5}{19}$

$\approx \boxed{0.263}$

eg

The data represent the results for a test for a certain disease. Assume one individual from the group is randomly selected. Find the probability of getting someone who tested negative, given that he or she did not have the disease.

The individual actually had the disease

	Yes	No
Positive	130	7
Negative	33	130

S: $P(\text{negative} \mid \text{no})$

$$= \frac{P(\text{negative and no})}{P(\text{no})}$$

$$= \frac{130}{137} \leftarrow 7 + 130$$

$$\approx \boxed{0.95}$$

Eg In an experiment, college students were given either four quarters or a $1 bill and they could either keep the money or spend it on gum. The results are summarized in the table. Find the probability of randomly selecting a student who kept the money, given that the student was given four quarters.

	Purchased Gum	Kept the Money
Students Given Four Quarters	34	15
Students Given a $1 Bill	13	27

S: $P(\text{kept the money} \mid \text{student given four quarters})$

$$= \frac{15}{49}$$

$$\approx \boxed{0.31}$$