Review: $\mu = \sum x\, p(x)$

$\sigma = \sqrt{\sum (x-\mu)^2 p(x)}$ ← a level, "a grade"

III. Expected Value

eg tossing a die,
eg gender on births
eg real insurance quote

In probability distribution, then mean μ is having a special name: the expected value. E.V.

Note: The expected value is only for probability distribution.

Furthermore, the expected value is not just the mean. It is also the average gain or loss of an event. That is

$$\boxed{E(x) = \sum x\, p(x)}$$ for two distribution, usually.

Property: If $E(x) > 0$, it is gain.

If $E(x) < 0$, it is loss.

eg Real life:

Time	Activity
10 hrs	\$
7 hrs	rest

$$E.V. = 10 \cdot \$ + 7 \cdot \text{rest}$$
$$= \boxed{\text{Daily life}}$$

← result

← I'd adjust

Eg. Find the expect value of the procedure below.

Bet	Earn
1	-1
2	-3

S: $E.V. = 1 \cdot (-1) + 2 \cdot (-3)$
$$= -1 + -6$$
$$= \boxed{-7}$$

eg Suppose you play a game with a biased coin. You play each game by tossing the coin once. P(heads) = 2/3 and P(tails) = 1/3. If you toss a head, you pay \$6. If you toss a tail, you win \$10. What is the expected value?

S:

	Money	Probability
head →	-6	$\frac{2}{3}$
tail →	10	$\frac{1}{3}$

$$E.V. = -6 \cdot \frac{2}{3} + 10 \cdot \frac{1}{3}$$
$$= -4 + \frac{10}{3}$$
$$\approx \boxed{-0.67} \text{ dollars}$$

eg Suppose you play a game of chance in which five numbers are chosen from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A computer randomly selects five numbers from zero to nine with replacement. You pay \$2 to play and could profit \$100,000 if you match all five numbers in order (you get your \$2 back plus \$100,000). Over the long term, what is your expected profit of playing the game?

S:

0–9: 10 choice, then 1st · 2nd · 3rd · 4th · 5th

Probability of winning: $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

$= 0.00001$

special here (pay attention) →

Money	Probability
100000	0.00001
−2	0.99999

Prob. of win: 0.00001

Prob. of lose: $1 - 0.00001 = 0.99999$

$E(x) = 100000 \cdot 0.00001 + -2 \cdot 0.99999$

$\approx \boxed{-1}$ dollar

Eg An insurance company estimates the probability of an earthquake in the next year to be 0.0013. The average damage done by an earthquake it estimates to be \$60,000. If the company offers earthquake insurance for \$100, what is their expected value of the policy?

S:

not free →

Money	Probability
\$60000 − \$100	0.0013
−\$100	1 − 0.0013

not free → $\$60000 - \$100$ | 0.0013

$\$60000-\$100$	0.0013
$-\$100$	$1-0.0013$

⇒

Money	Probability
59900	0.0013
-100	0.9987

$$E(X) = 59900 \cdot 0.0013 + -100 \cdot 0.9987$$
$$= \boxed{-22} \text{ dollars}$$

Eg A roulette has 18 green trays, 18 red trays, and 1 white tray for the ball to land in. The casino takes your bet of \$5 that the ball will land in a green tray. The casino will pay you \$10 if the ball lands on the color green. The probability of winning by betting on green is 18/37. What is the expected value for the bettor?

S:

Total: $18+18+1=37$

$\$10-\$5 \rightarrow$

Money	Probability
5	$\frac{18}{37}$
-5	$\frac{19}{37}$

← $1 - \frac{18}{37} = \frac{19}{37}$

$$E.V. = 5 \cdot \frac{18}{37} + -5 \cdot \frac{19}{37}$$
$$\approx \boxed{-0.14} \text{ dollar}$$

Eg Now suppose you bet \$1 on #12. If ball lands on #12, you get \$35. Otherwise you lose \$1. What is the new expected value going to be?

S:

\$35 − \$1 →

Money	Probability	
34	$\frac{1}{37}$	← get #12
−1	$\frac{36}{37}$	← not get #12

$$E.V. = 34 \cdot \frac{1}{37} + -1 \cdot \frac{36}{37}$$

$$\approx \boxed{-0.05} \text{ dollar}$$