V. Poisson Distribution

another kind of binomial distribution

small prob.

num. of occurance is small

Simeon Poisson's work.

A <u>Poisson distribution</u> is a discrete probability distribution that applies to occurences of <u>some events</u> over a specified interval.

Requirement:

1. The variable x is a number of occurance <u>in some interval</u>.
2. The occurance has to be <u>random and independent</u>.
3. The occurances is <u>uniformly distributed</u> over the <u>interval</u> being used.

be aware

Examples:

"is strictly less than a number"

< 7

eg Spam calls.

eg Ivy school's adimitted in your high school.

eg Rare disease.

⋮

<u>Formula</u>: The prob. of the event occurring x times over an interval is given by

$$\boxed{P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}}$$

μ is λ in TI-84

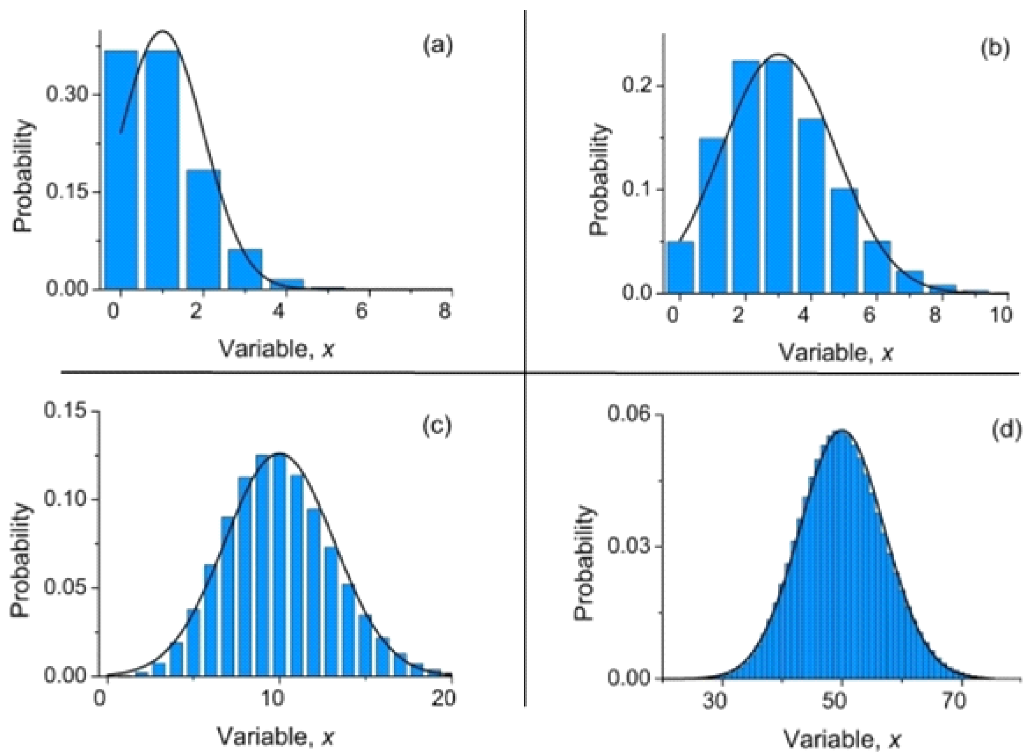
$$\boxed{P(x) = \frac{\mu^{x} \cdot e^{-\mu}}{x!}},$$

where $e \approx 2.72$ and μ = mean of the occurance of event in the intervals.

Note: The number of occurance is <u>small</u>. $x < 7$

(The $P(x)$ is small, as well.)

Graph



Eg There have been 652 Atlantic hurricanes during the 118-year period starting in 1900. Assume that the Poisson distribution is a suitable model.

a. Find μ, the mean number of hurricanes per year.

b. Find the probability that in a randomly selected year, there are exactly 6 hurricanes. Find P(6), where P(x) is the probability of x Atlantic hurricanes in a year.

c. In this 118-year period, there were actually 16 years with 6 Atlantic hurricanes.

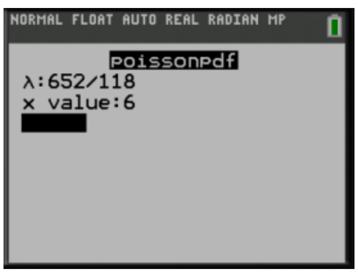
How does this actual result compare to the probability found in part (b)? Does the Poisson distribution appear to be a good model in this case?

S: a. $\mu = \dfrac{652 \leftarrow \text{hurricane}}{118 \leftarrow \text{years}} \approx \boxed{5.5}$

b. $P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$

$P(6) = \dfrac{5.5^6 e^{-5.5}}{6!} \approx \boxed{0.157}$

TI-84: 2nd → distr → C: Poissonpdf(← D for newer TI-84



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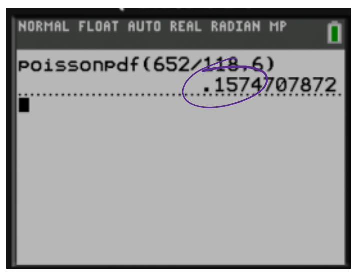

Eg There have been 652 Atlantic hurricanes during the 118-year period starting in 1900. Assume that the Poisson distribution is a suitable model.
~~a. Find μ, the mean number of hurricanes per year.~~
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c. In this 118-year period, there were actually 16 years with 6 Atlantic hurricanes. How does this actual result compare to the probability found in part (b)? Does the Poisson distribution appear to be a good model in this case?

S: With 6 hurriances for 16 years, compare to 118 years. We have μ is 5.5. Then, $\frac{16}{6} \approx 2.7 < \mu = 5.5$. The occurance is small for that 16 years. Therefore, the Poisson distribution works well.

Poisson → Binomial :

When the number of occurance is over 100 and the probability is small, the Poisson distribution would become binomial distribution. Therefore, for $n \geq 100$, the mean for Poisson distribution is $\boxed{\mu = np}$.

Eg In the Maine Pick 4 game, you pay 50¢ to select a sequence of four digits (0–9), such as 1377. If you play this game once every day, find the probability of winning at least once in a year with 365 days.

S: We have $n = 365$, and $\underset{10}{0\text{-}9} \cdot \underset{10}{0\text{-}9} \cdot \underset{10}{0\text{-}9} \cdot \underset{10}{0\text{-}9}$'s probability $\frac{1}{\_\ \_\ \_\ \_}$

$= \frac{1}{10000} = 0.0001$. Then, we have

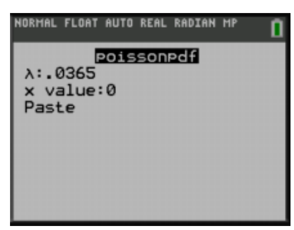
$$\mu = 365 \cdot 0.0001 = 0.0365$$

Then, $P(\text{at least } 1) = P(1 \text{ or more})$

$$= 1 - P(0)$$

$$= 1 - 0.964$$

$$= \boxed{0.036}$$



Eg Leah receives about six telephone calls between 8 a.m. and 10 a.m.

What is the probability that Leah receives more than one call in the next 15 minutes?

S: We have $x = 0, 1, 2, 3$, interval is 15 minutes

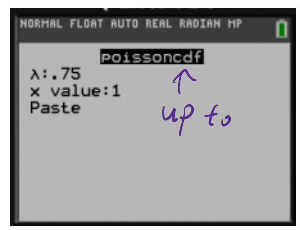
$$15 \text{ minutes} = 15 \text{ minutes} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$= 0.25$$

From 8 am – 10 am: $\frac{2}{0.25} = 8$ intervals

then, $\mu = 6 \cdot \frac{1}{8} = 0.75$,

$$P(x > 1) = 1 - \underbrace{\left(P(0) + P(1)\right)}_{\text{up to } 1} \rightarrow$$



$$= 1 - 0.827$$

$$= \boxed{0.173}$$