VI. Hypothesis Testing on $\underline{\underline{\text{Mean}}}$

← today: $H_0: \mu = 22$
$H_1: \mu > 22$

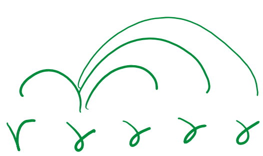
For testing the mean, we would have the sample calculation. The size n can be any, because of the Student t-distribution. Then, the t-test is used because it gives better results than the regular z-test.

n = sample size

$\bar{x}$ = sample mean ← new data values

μ = population mean ← H_0's old mean

Test Statistics:



$$\boxed{t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}}$$

← s is the sample's std. dev.

with $\boxed{d.f. = n-1}$

eg. Listed below are the measured radiation emissions (in W/kg) corresponding to a sample of cell phones.

Use a 0.05 level of significance to test the claim that cell phones have a mean radiation level that is less than 1.00 W/kg.

0.38	0.55	1.54	1.55	0.50	0.60	0.92	0.96	1.00	0.86	1.46

S:

$H_0: \mu = 1$

$H_1: \mu < 1$

11 of them

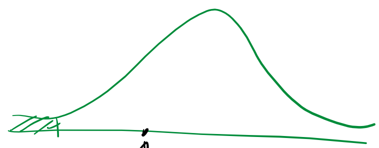
$\underline{\underline{n = 11}}$

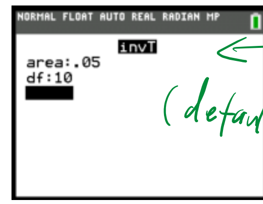
$H_1: \mu < 1$

$n = 11$

$\alpha = 0.05$, with $<$, and

$d.f. = n - 1 = 11 - 1 = 10$





invT
(default is left tail)

$\bar{x} = TI\text{-}84 \approx 0.94$

$s = TI\text{-}84 \approx 0.42$

Then,

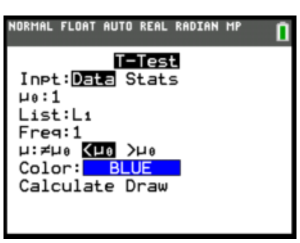
$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{0.94 - 1.00}{\frac{0.42}{\sqrt{11}}} \approx -0.47$$

Since $t = -0.47 > C.V. = -1.81$

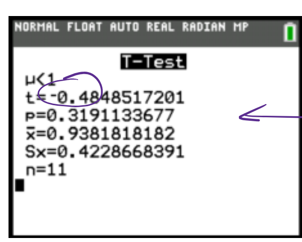
H_1 fails to reject H_0.

Therefore, there is not enough evidence to support the claim that cell phones have a mean radiation level that is less than 1.00 w/kg.

TI-84: Stat → TESTS → 2: T-Test...



→



$p \approx 0.32 > \alpha = 0.05$

eg Listed below are the lead concentrations in µg/g measured in different traditional medicines. Use a 0.05 significance level to test the claim that eg Listed below are the lead concentrations in μg/g measured in different traditional medicines. Use a 0.05 significance level to test the claim that the mean lead concentration for all such medicines is less than 15 μg/g. Assume that the sample is a simple random sample.

13.5 11.5 14.5 4 3 22 7 16 6.5 2.5

$n = 10$

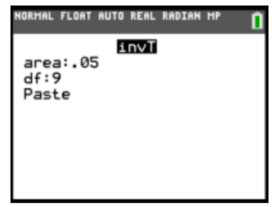
S:

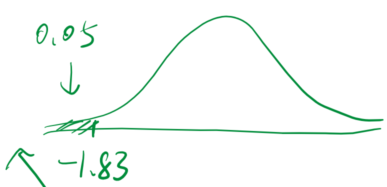
$H_0: \mu = 15$

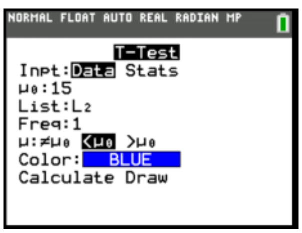
$H_1: \mu < 15$

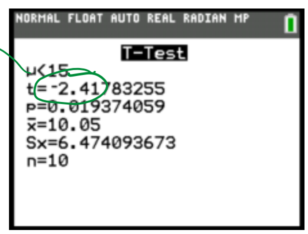
$\alpha = 0.05$ with $<$, and

$d.f. = n - 1 = 10 - 1 = 9$









Since $t = -2.42 < C.V. = -1.83$,

H_1 rejects H_0.

Thus, there is enough evidence to support the claim that the mean lead concentration for the medicine is less than 15 μg/g.

$n =$

eg In a test of the effectiveness of garlic for lowering cholesterol, 49 subjects were treated with raw garlic. Cholesterol levels were measured before and after the treatment. The changes (before minus after) in their levels of LDL cholesterol (in mg/dL) have a mean of 0.6 and a standard deviation of 17.6. Use a 0.10 significance level to test the claim that with garlic treatment, the mean change in LDL cholesterol is greater than 0. What do the results suggest about the effectiveness of the garlic treatment? Assume that a simple random sample has been selected.

S:

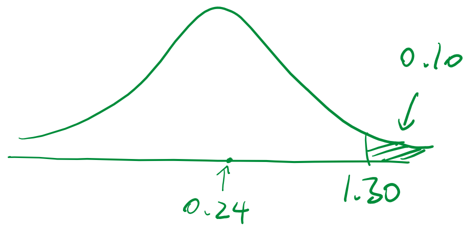
$H_0: \mu = 0$

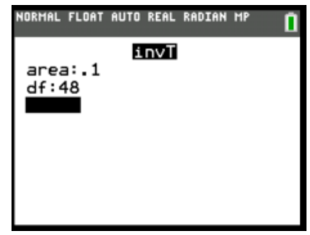
$H_1: \mu > 0$

$n = 49$

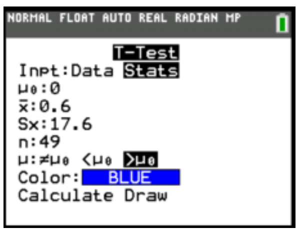
$\alpha = 0.10$ with $>$, and

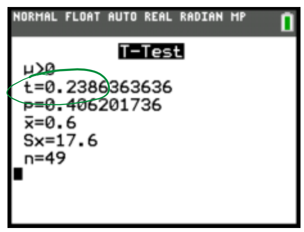
$d.f. = n-1 = 49-1 = 48$





$\bar{x} = 0.6$, $s = 17.6$





Since $t = 0.24 < C.V. = 1.30$,

H_1 fails to reject H_0. ← really fail; H_0 steadily retains.

Thus, there is not enough evidence to support the claim that the mean of changes in garlic LDL cholesterol level is greater than 0.

eg Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples ten statistics students and obtains the scores 65; 65; 70; 67; 66; 63; 63; 68; 72; 71. Perform a hypothesis test using a 5% level of significance.

10 of them

$n = 10$

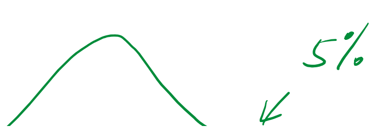
S:

$H_0: \mu = 65$

$H_1: \mu > 65$

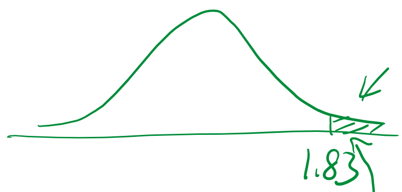
$\alpha = 0.05$ with $>$ and

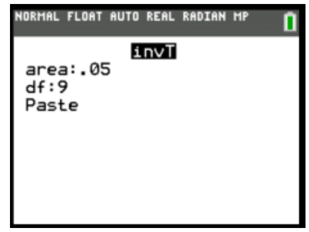
$d.f. = n-1 = 10-1 = 9$

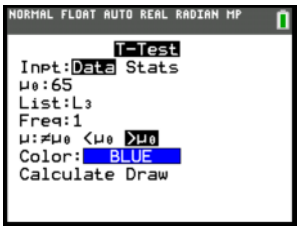


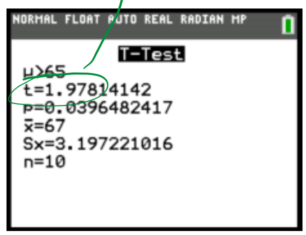
$\alpha = 0.05$ with 5 and

$d.f. = n-1 = 10-1 = 9$









Since $t = 1.98 > C.V. = 1.83$,

H_1 rejects H_0.

Therefore, there is sufficient enough evidence to support the claim that the mean test score is greater than 65.

An example from a different instructor:

2. Calcium is essential to tree growth. In 1990, the concentration of calcium in precipitation in Chautauqua, New York, was 0.11 milligram per liter (mg/L). A random sample of 10 precipitation dates in 2018 results in the following data:

0.065	0.087	0.183	0.120	0.126
0.070	0.234	0.262	0.313	0.108

already produced

Source: National Atmospheric Deposition Program.

A box plot suggests there is no evidence of strong skewness and no outliers. Does the sample evidence suggest that calcium concentrations have changed since 1990? Use the $\alpha = 0.05$ level of significance. Use $\bar{x} = 0.1568$, $s = 0.0867$, and P-value = 0.0610.

S:

$\boxed{\text{Step 1}}$ $H_0: \mu = 0.11$ mg/L
$H_1: \mu \neq 0.11$ mg/L

two-tailed test

$\boxed{\text{Step 2}}$ $\alpha = 0.05$

$\boxed{\text{Step 3}}$ $t_0 = \dfrac{0.1568 - 0.11}{\frac{0.0867}{\sqrt{10}}} = 1.707$

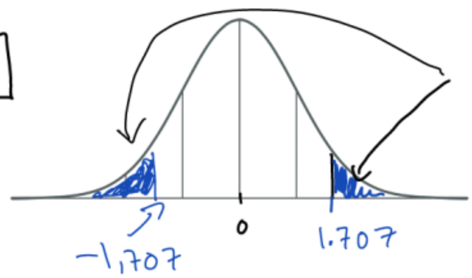
S:

$\boxed{\text{Step 1}}$ $H_0: \mu = 0.11$ mg/L ← two-tailed test
$H_1: \mu \neq 0.11$ mg/L

$\boxed{\text{Step 2}}$ $\alpha = 0.05$

$\boxed{\text{Step 3}}$ $t_0 = \dfrac{0.1568 - 0.11}{\frac{0.0867}{\sqrt{10}}} = 1.707$

$\boxed{\text{Step 4}}$



P-value = 0.061

$\boxed{\text{Step 5}}$ P-value = 0.061 > 0.05 , do not reject H_0

$\boxed{\text{Step 6}}$ There is not sufficient evidence to indicate that the calcium concentration in rainwater in Chautauqua, N.Y. has changed since 1990.